

Window Image Reflection Removal by Blind Image Separation

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Abstract

A novel second order statistics approach to blind image separation has been deployed to remove reflection image from window glass images. The reflection image and the image from behind the window glass are considered as blind sources. Since sources are uncorrelated, their low frequency as well as high frequency halfbands are uncorrelated too. It is shown that the un-mixing matrix is obtained by simultaneous diagonalization of covariance matrices of low and high halfbands of the mixtures. The proposed technique has been compared with AMUSE [1] and SOBI [2] as two other well known second-order statistics BSS techniques. Three different numerical metrics approves the efficiency dominance of the proposed technique.

Keywords: Reflection removal, Blind image separation, second-order statistics, covariance matrix, subband linear filters.

1 Introduction

Although images normally do not mix together in a well engineered system, blind image separation is a problem encountered in various applications such as document image restoration, astrophysical component separation, analysis of functional Magnetic Resonance Imaging and removal of spurious reflections. In taking photograph from the objects behind the window glass, the taken photo is a mixture of the image of the objects behind the glass and the reflection image of the objects in front of the glass. Such event happens in window shopping, aquarium photography and any other photography with a glass window between camera and objects. Also in photography of inside the water from above of water surface, the photo is mixture of inside the water and reflection of the sky and clouds appeared on the water surface.

Blind source separation (BSS) [3, 4] is a well known technique to recover multiple original sources from their mixtures. While K unknown source signals upon transmission through a medium have been mixed to-

gether and mixture signals are collected by M sensors, BSS try to blindly separate the sources from mixed observed data without any prior information of the source signals and mixing process. In mathematical model, the $M \times 1$ vector \mathbf{x} of observed signals $\mathbf{x}(t) = (x_1(t) \ x_2(t) \ \cdots \ x_M(t))^T$ is multiplication of $K \times 1$ vector \mathbf{s} of unknown source signals $\mathbf{s}(t) = (s_1(t) \ s_2(t) \ \cdots \ s_K(t))^T$ by unknown mixing matrix $\mathbf{A} \in R^{M \times K}$:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{n}(t)$ is a possible additive noise. Given only the observation signals $\mathbf{x}(t) = (x_1(t) \ x_2(t) \ \cdots \ x_M(t))^T$, the solution of BSS problem seeks for the best $\mathbf{W} \in R^{K \times M}$ as un-mixing matrix to extract signals as much as possible close to unknown source signals as follows:

$$\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t) \quad (2)$$

where $\mathbf{y}(t)$ is $K \times 1$ vector of extracted signals $\mathbf{y}(t) = (y_1(t) \ y_2(t) \ \cdots \ y_K(t))^T$. \mathbf{W} is obtained in the way that the estimated sources to be spatially uncorrelated or as independent as possible, in the sense of minimizing various functions that measure independence [5-9]. This measure is based upon second-order statistics (SOS) or higher-order statistics (HOS). Many second-order BSS approaches have been presented in the literature [10-18]. One of the well known second-order BSS methods is an algorithm for multiple unknown signals extraction (AMUSE) [1]. AMUSE converts the problem of blind identification of mixing matrix to simultaneous diagonalization (SD) problem of two covariance matrices of mixtures and their delayed signals. The other well known second-order BSS method is second-order blind identification (SOBI) algorithms [2]. SOBI employs joint diagonalization of covariance matrix of mixtures and covariance matrices of different delayed mixtures to obtain un-mixing matrix.

In this paper, the reflection over window glass is blindly separated from the image of the objects behind the glass by using a BSS approach by deploying second-order statistics of the sources in two different views at the same time. The proposed BSS is based on assumption that sources are mutually uncorrelated. When sources are uncorrelated, their subbands

obtained by linear filters are uncorrelated too. The simultaneous diagonalization (SD) of covariance matrices of the low halfbands and high halfbands of image mixtures has been deployed to remove glass reflection of window images. The proposed BSS method has been compared with AMUSE and SOBI.

The rest of the paper is as follows. In section 2, the BSS by SD of high and low halfbands has been described. Experimental results and discussions are presented in section 3. Finally, Section 4 concludes the paper.

2 BSS by simultaneous diagonalization of low and high halfbands

The proposed BSS is based on second-order statistics of low and high halfbands of the uncorrelated sources. In order to extend the theory of the proposed BSS, first we express some properties of responses of a linear filters to uncorrelated signals. Consider a vector of signals $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$, and the vector of responses of a linear filter to them as $\bar{\mathbf{s}}(t) = [\bar{s}_1(t), \bar{s}_2(t), \dots, \bar{s}_K(t)]^T$. For responses of a linear filter to a vector of signals, the following properties can be obtained;

- **Property 1** If signals $\mathbf{s}(t)$ are mutually uncorrelated, then $\bar{\mathbf{s}}(t)$ the response signals of a linear filter to them are also mutually uncorrelated.

$$E[s_i s_j] = 0, \quad i \neq j \implies E[\bar{s}_i \bar{s}_j] = 0 \quad i \neq j \quad (3)$$

So, the covariance matrix of the response signals is a diagonal matrix;

$$C_{\bar{\mathbf{s}}} = \text{diag}(E[\bar{s}_1 \bar{s}_1], E[\bar{s}_2 \bar{s}_2], \dots, E[\bar{s}_K \bar{s}_K])$$

- **Property 2** Let $\bar{\mathbf{y}}(t)$ and $\bar{\mathbf{x}}(t)$ are respectively responses of a linear filter to $\mathbf{y}(t)$ and $\mathbf{x}(t)$. If $\mathbf{y}(t) = \mathbf{U}\mathbf{x}(t)$, $\mathbf{U} \in R^{K \times M}$. Then, we have

$$\bar{\mathbf{y}}(t) = \mathbf{U}\bar{\mathbf{x}}(t) \quad (4)$$

$$\text{and} \quad C_{\bar{\mathbf{y}}} = \mathbf{U}C_{\bar{\mathbf{x}}}\mathbf{U}^T \quad (5)$$

where $C_{\bar{\mathbf{x}}}$ and $C_{\bar{\mathbf{y}}}$ are respectively covariance matrices of $\bar{\mathbf{x}}(t)$ and $\bar{\mathbf{y}}(t)$.

Let's remember the above properties and go back to the BSS problem. Vector of M mixture signals $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$ has been observed by M sensors. BSS looks for the best estimate of un-mixing matrix $\mathbf{W}_{K \times M}$ in order to recover the vector of K unknown source signals $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_K(t)]^T$ by Eq.(2).

$$\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t) \approx \mathbf{s}(t) \quad (6)$$

Consider $\bar{\mathbf{y}}(t)$ and $\tilde{\mathbf{y}}(t)$ are respectively low and high halfbands of the recovered signals by \mathbf{W} . $\bar{\mathbf{y}}(t)$ and $\tilde{\mathbf{y}}(t)$ have been obtained by a linear lowpass filter and a linear highpass filter, respectively. Throughout this section the following assumptions are made unless stated otherwise:

- **Assumption 1** : The mixing matrix $\mathbf{A} \in R^{M \times K}$ is a full column rank matrix with $M \geq K$, and it is the same for all frequency contents of signals.
- **Assumption 2** : Sources are mutually uncorrelated with different autocorrelation functions.

$$E[s_i s_j] = 0, \quad i \neq j \quad (7)$$

$$E[s_i s_i] \neq E[s_j s_j] \quad i \neq j \quad (8)$$

- **Assumption 3** : Autocorrelation functions of low halfbands of the sources are different from autocorrelation functions of their high halfbands.

$$E[\bar{s}_i \bar{s}_i] \neq E[\tilde{s}_i \tilde{s}_i] \quad (9)$$

If \mathbf{W} is the un-mixing matrix, based on assumption 2 recovered source signals are mutually uncorrelated. Also, since low and high halfbands are obtained by linear filters, from property 1, the covariance matrices of $\bar{\mathbf{y}}(t)$ and $\tilde{\mathbf{y}}(t)$ are diagonal matrices;

$$C_{\bar{\mathbf{y}}} = E[\bar{\mathbf{y}}\bar{\mathbf{y}}^T] \quad (10)$$

$$= \text{diag}(E[\bar{y}_1 \bar{y}_1], E[\bar{y}_2 \bar{y}_2], \dots, E[\bar{y}_K \bar{y}_K]),$$

$$\tilde{C}_{\bar{\mathbf{y}}} = E[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^T] \quad (11)$$

$$= \text{diag}(E[\tilde{y}_1 \tilde{y}_1], E[\tilde{y}_2 \tilde{y}_2], \dots, E[\tilde{y}_K \tilde{y}_K]),$$

where $C_{\bar{\mathbf{y}}}$ and $\tilde{C}_{\bar{\mathbf{y}}}$ are covariance matrices of $\bar{\mathbf{y}}(t)$ and $\tilde{\mathbf{y}}(t)$. Furthermore, since from Assumption 3 the low and high halfbands of the sources have different autocorrelation functions, both $C_{\bar{\mathbf{y}}}$ and $\tilde{C}_{\bar{\mathbf{y}}}$ are non-zero distinct matrices. Taking into account $\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t)$ and considering the property 2, we have

$$C_{\bar{\mathbf{y}}} = \mathbf{W}C_{\bar{\mathbf{x}}}\mathbf{W}^T \quad (12)$$

$$\tilde{C}_{\bar{\mathbf{y}}} = \mathbf{W}\tilde{C}_{\bar{\mathbf{x}}}\mathbf{W}^T \quad (13)$$

where $C_{\bar{\mathbf{x}}}$ and $\tilde{C}_{\bar{\mathbf{x}}}$ are covariance matrices of $\bar{\mathbf{x}}$ and $\tilde{\mathbf{x}}$. Since $C_{\bar{\mathbf{y}}}$ and $\tilde{C}_{\bar{\mathbf{y}}}$ are distinct matrices, it follows from Eqs.(12) and (13) that $C_{\bar{\mathbf{x}}}$ and $\tilde{C}_{\bar{\mathbf{x}}}$ are distinct matrices too. Moreover, Eqs.(12) and (13) convey that simultaneous diagonalization of two distinct matrices $C_{\bar{\mathbf{x}}}$ and $\tilde{C}_{\bar{\mathbf{x}}}$ estimates the un-mixing matrix \mathbf{W} up to its re-scaled and permuted version.

For the sake of simplicity, the problem of simultaneous diagonalization is converted to generalized eigenvalue decomposition (GEVD) [1] as follows. Since, in

the case of separation, \bar{C}_{yy} and \tilde{C}_{yy} are distinct diagonal matrices, their multiplication $R_{yy} = \bar{C}_{yy}\tilde{C}_{yy}$ is a diagonal matrix;

$$\begin{aligned} R_{yy} &= \bar{C}_{yy}\tilde{C}_{yy} \\ &= \text{diag} (E[\bar{y}_1\bar{y}_1]E[\tilde{y}_1\tilde{y}_1], E[\bar{y}_2\bar{y}_2]E[\tilde{y}_2\tilde{y}_2], \dots \\ &\quad \dots, E[\bar{y}_K\bar{y}_K]E[\tilde{y}_K\tilde{y}_K]) \end{aligned} \quad (14)$$

On the other hand, from Eqs. (12) and (13)

$$R_{yy} = [W\bar{C}_{xx}W^T][W\tilde{C}_{xx}W^T] \quad (15)$$

$$= W\bar{C}_{xx}[W^TW]\tilde{C}_{xx}W^T \quad (16)$$

If we convert the problem to GEVD, it will allow us to make the assumption of orthogonality as follows:

- **Assumption 4** : The un-mixing matrix W can be composed of orthogonal separating vectors.

By applying Assumption 4 ($W^TW = I$) to Eq.(15), we have

$$\bar{C}_{xx}\tilde{C}_{xx} = W^{-1}R_{yy}W. \quad (17)$$

Since R_{yy} is a diagonal matrix, Eq.(17) conveys that:
The un-mixing matrix W is the matrix which diagonalizes $\bar{C}_{xx}\tilde{C}_{xx}$.

Based on the result of Eq.(17) and by using diagonalization theorem [19], the un-mixing matrix W is obtained as follows.

The un-mixing matrix W is composed of eigenvectors of $\bar{C}_{xx}\tilde{C}_{xx}$.

Since, $\bar{C}_{xx}\tilde{C}_{xx}$ is a symmetric square matrix, its eigenvectors are orthogonal, and Assumption 4 is approved.

The algorithm of blind source separation by simultaneous diagonalization of two covariance matrices of responses of two different linear filters to the mixture signals can now be described by the following steps .

The Algorithm Outline:

- **Step 1** : Obtain $\bar{x}(t)$ and $\tilde{x}(t)$ the responses of two different linear filters \bar{F} and \tilde{F} to observed mixture signals $x(t)$.
- **Step 2** : Compute \bar{C}_{xx} and \tilde{C}_{xx} the covariance matrices of $\bar{x}(t)$ and $\tilde{x}(t)$ as well as their multiplication $R_{xx} = \bar{C}_{xx}\tilde{C}_{xx}$.
- **Step 3** : Find the generalized eigenvector matrix V for the generalized eigenvalue decomposition;

$$R_{xx}V = VD \quad (18)$$

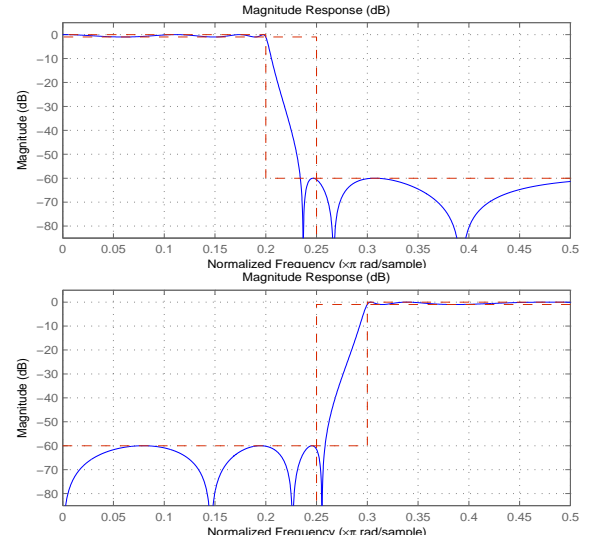


Figure 1: Frequency responses of the deployed lowpass filter (top) and highpass filter (bottom).

- **Step 4** : The un-mixing matrix up to its re-scaled and permuted version is given by $W = V$ on the condition that all eigenvalues are real and distinct.

The quality of separation depends on how much the assumptions 2 and 3 are strong. The more different autocorrelation functions of the low halfbands from autocorrelation functions of the high halfbands results in better separation result.

3 Results and Discussion

To empirically evaluate potential usefulness of the proposed method, some evaluation tests. We have used IIR elliptic lowpass and highpass filters to obtain low and high halfbands of the mixtures, respectively. In this simulation, in all of the cases the lowpass filter is a minimum order filter with a normalized passband frequency of 0.2, a stopband frequency of 0.25, a passband ripple of 1 dB, and a stopband attenuation of 60 dB, and the highpass filter is also a minimum order filter with a normalized passband frequency of 0.3, a stopband frequency of 0.25, a passband ripple of 1 dB, and a stopband attenuation of 60 dB. The above mentioned frequencies of the filters specifications are normalized with respect to sampling frequency of the signals. Since, from Niquist sampling theorem the bandwidth of the signals is maximally considered as half the sampling frequency, to obtain halfbands of the mixtures, the optimum stop frequency of both lowpass and highpass filters is $\frac{f_s}{4}$, where f_s is the sampling frequency. Fig. 1 shows frequency responses of the deployed lowpass and high pass filters.

We have compared the performance of the proposed method with two well known second-order BSS al-

gorithms, AMUSE (Algorithm for Multiple Unknown Signals Extraction) [1] and SOBI (Second Order Blind Identification) [2]. Both AMUSE and SOBI jointly exploit the second order statistics (correlation matrices for different time delays) and temporal structure of sources. In this simulation, the optimal delay is 1 time sample in AMUSE BSS, and number of matrices which are jointly diagonalized at each run of SOBI algorithm is 4.

We have evaluated the method with a real experiment. Two images have been taken of the same view of street from inside the window glass of a room. Both images belong to the same time of the day in different light conditions of the room. Figure 2 (last page) shows the taken photos and reflection removed photos as well as extracted reflection by the above mentioned BSS methods. There exist many possible performance criterion in the literature [20]. Most of evaluation methods are by using sources or global matrix (multiplication of mixing matrix and demixing matrix $\mathbf{G} = \mathbf{A}\mathbf{W}$). But here, since, mixing matrix and sources are not available, it is not possible to use most of evaluation methods mentioned in literature. We have just real mixed observed images and separated ones. In order to evaluate the separation performance, we have used three numerical indexes for measuring similarity of recovered images obtained by each method. We have measured the correlation, mean absolute distance (MAD) and mean squared distance (MSD) between two recovered images obtained by each method. Each of the less correlation, the more MAD and the more MSD individually shows the less similarity of recovered images as well as the better performance of separation. Table.1 shows these numerical evaluations result in reflection removal from real reflected window images of figure.2. As it is seen all the three indexes approve higher performance of the proposed method.

3.1 Robustness of reflection removal

In addition to real experiment of Fig.2, the robustness of the method to noise and different images and reflections has been artificially investigated too. To aim this, we have used a set of 15 different gray-scale images as source images and reflections. They were randomly se-

Table 1: Performance comparison of the proposed method with AMUSE and SOBI in reflection removal of real window glass images of figure 2.

	BSS Methods		
	AMUSE	SOBI	Proposed
Correlation :	0.8704	0.8707	0.8583
MAD :	4.5892e + 03	4.5569e + 03	6.2269e + 03
MSD :	30.5779	30.5089	36.7917

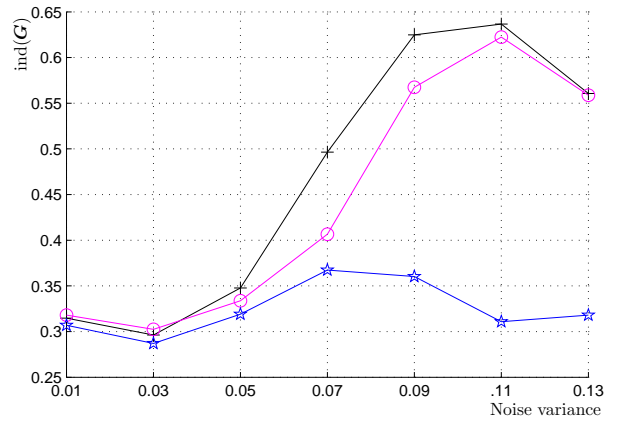


Figure 3: The averaged $ind(\mathbf{G})$ Vs. variance of noise in separation mixed images by AMUSE (plus - black), SOBI (circle - pink), And SD of low and high halfbands (star - blue).

lected and mixed each other with full rank random matrix. The mixing matrix was the same for all methods. At each noise level, the performance of the methods have been compared by averaged result of 200 Mont Carlo runs. Since the mixing matrix was available for evaluation, we used the $ind(\mathbf{G})$ [20] the index based on global matrix. The global matrix is obtained by multiplication of mixing and de-mixing matrices $\mathbf{G} = \mathbf{W}\mathbf{A}$. This performance index is as follows

$$ind(\mathbf{G}) = \frac{1}{K(K-1)} \left[\sum_i^K \left(\sum_j^K \frac{|G_{i,j}|^2}{\max_l |G_{i,l}|^2} - 1 \right) + \sum_j^K \left(\sum_i^K \frac{|G_{i,j}|^2}{\max_l |G_{l,j}|^2} - 1 \right) \right] \quad (19)$$

where K is dimension of \mathbf{G} . Indeed, this non-negative index is zero in the case of perfect separation, and its the smaller value indicates the more proximity to the desired solutions.

Fig.3 shows the averaged $ind(\mathbf{G})$ Vs. variance of noise in separation of gray-scaled images. As it is seen, the proposed method is less affected by noise level than the others and it has the highest performance in comparison to the others.

4 conclusion

In this paper, the reflection over window glass images has been removed by simultaneous diagonalization of low and high halfbands of the mixtures. Decomposing the mixtures to the set of low subband and high subband mixtures enables us to have two different views of the mixtures. The simultaneous diagonalization in two different views results in higher performance of separation. The proposed second order technique has been compared with AMUSE [1] and SOBI [2] in a real experiment. Furthermore, it robustness to noise and dif-

ferent sources has been compared to the others. Three different numerical metrics approves dominance of the proposed BSS to the others in removing the reflection from window glass images.

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Images
with
reflections



AMUSE



SOBI



Proposed



Figure 2: Real window images with reflection (1st row), separated window images and reflections by AMUSE (2nd row), by SOBI (3rd row) and by Simultaneous Diagonalization of low and high halfbnds (bottom row).