Stabilization of MIMO Time Delay System Using Hybrid Controller of Internal Model Control and Feedforward Control

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Abstract

In this paper, we aim to stabilize the unstable system by loop shaping method. Moreover after stabilizing the system we need to modify the system which is unstable due to time-delay element by Internal Model Controller with feed forward considering the uncertainty of control object and time-delay el. Time-delay will occur during the long distance communication. Therefore, when control object is located in distance, the transmitted reference signal will be delayed. For this reason even though we stabilize the control plant by loop-shaping, it will be an unstable system due to time-delay elements. In this paper we consider an unstable control plant such as HIMAT. Generally, HIMAT is unmanned aircraft which is required be controlled from distance by communication system that means in this system we have time-delay element. Our aim is to stabilize the closed loop system with Internal Model controller and realize the better performance of system using feed forward control.

Key Words: Time-Delay System, Robust Control, Internal Model Control, Feedforward Control

1. Introduction

In this research, we propose control of the time-delay system by using H^{∞} Controller, IMC (Internal Model Controller) and FFC (feedforward Controller). Until now there were many schemes to consider about the designing controller for systems which including time-delay element. For example the classical way is PID (Proportional-Integral-Derivative) [6]. However, this scheme, do not warranty the stability for large time-delay. On the other hand, LQI (Linear Quadratic Integration) [2], [10], [11], [12] method is modern control scheme and it warranties the stability even for a long time-delay. However, for MIMO system, it is very complicated to solve the Ricatti equation [14], [15], [16] to design the optimum controller. Therefore, we consider combination of IMC and FFC method [18], [19], [20] to stabilize the closed loop system including time-delay element. Time-delay will happen during utilization of the long distance communication. The application of the long distance communication is an important issue in aerospace engineering. When we have a control plant in the distance, the transmitter's signal will be delayed. Therefore the received signal at the control plant will also be delayed. Moreover, the feedback signal to transmitter location (in order to compare the output signal and reference signal) will also be delayed. In this case, we have a round trip delay, one delay is to reach the control plant and another delay is to receive the feedback signal for comparison with the reference signal. Moreover, generally the control plant of the system usually is an unstable system. Therefore, we have to consider the stabilization of the control plant. In this paper, we suggest the loop shaping method with H^{∞} controller. By loop shaping method, we can make the control plant stable. Though control plant will be unstable due to time-delay element. Therefore, after stabilizing the control plant, we use combination of IMC and FFC method to make the system stable which was unstable due to time-delay element with good performance. Also, we considered the uncertainty of control plant and time-delay element which it estimated and approximated by using Pade approximation. Here loop shaping method makes the open loop singular value to make the same as desirable loop shape considering stability robustness and performance. IMC method is minimizing the coefficient of external disturbance of output signal by H^2 norm and FFC has role of making the system's performance better as much as possible by tuning the filter. Therefore, by this design, problems of the system instability and uncertainty have been overcome.

2. The Theory of Time Delay System and Back ground of Research

In this research we consider a round trip time-delay system which means one delay element to reach the control plant and another delay element to feedback the output signal in order to compare with reference signal. Figure.1 shows the block diagram of a round trip time-delay system without controller.

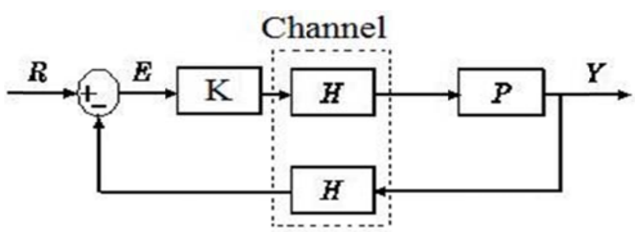

Fig. 1: Time-delay system

Here “Y”,”H”,”R”,”P” and “E” are the output signal of the system, time-delay element, reference signal, control object and error signal of the system, respectively. Through figure 1 it is clear that the sensitivity (S) and complementary sensitivity (T) are obtained as follows:

$$S = (I + PKH^2)^{-1} \qquad (1)$$

$$T = (I + PKH^2)^{-1} PKH \qquad (2)$$

Generally, in feedback control system, by adding some controller such as “K” which is designed corresponding to control plant, we can make the system stable or decrease the error. In classical control usually PID controller and in modern control integrator operation and optimum gain such as LQI method are used. As a result, for PID controller if time-delay is large, system could not preserve the stability. But for Modern scheme LQI method we could make the system stable without error. However, for high dimension and MIMO system we could not design the optimum controller because of complexity of solving the Ricatti equation. In this research, the system is already an unstable and contains with time-delay elements. Therefore, we propose the IMC, FFC method and loop shaping method with H^∞ controller which the whole system is structured by hybrid system controllers. At first, loop shaping with H^∞ controller is designed corresponding to control plant to make it stable. IMC and FFC is designed corresponding to uncertainty of time-delay element and control plant in order to stabilize the closed loop system.

3. *Loop Shaping Method with H^∞ Controller*

For the Process of stabilizing the closed loop system is at first, it is required to stabilize control object. Therefore, in this case we design H^∞ controller by loop shaping method. For loop shaping method, let us design a stabilizing feedback controller to optimally shape the open loop frequency response of a MIMO feedback control system to match as closely as possible to a desired loop shape. Controller has the property that it shapes the open loop so that it matches the frequency response of target desired loop shape as closely as possible.

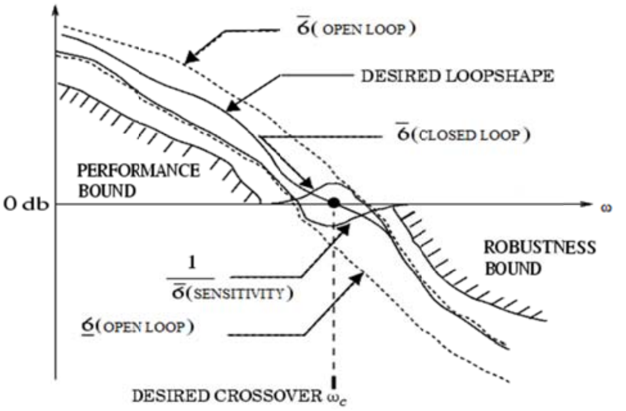

Fig.2: The singular value specifications on open loop, sensitivity, and closed loop

Through figure 2, it shows that the desired loop shape, in low frequencies has high gain and in high frequencies has low gain. This is because of in low frequencies if open loop’s singular values have high gain the output signal has better performance and if it has low gain in high frequencies it is robust to disturbance, noise and uncertainty. Also for complementary sensitivity singular values it is desirable to have low gain in high frequencies in order to robust to noise and uncertainty. Derivation of loop-shape controller is shown as follows:

Since stability margin (SM) is inverse of transfer function between $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \rightarrow \begin{bmatrix} y_s \\ u_s \end{bmatrix}$ of infinity norm that is:

Fig .3: Standard feedback interconnection

$$\begin{bmatrix} y_s \\ u_s \end{bmatrix} = \begin{bmatrix} -(I-PC)^{-1} & (I-PC)^{-1}P \\ -C(I-PC)^{-1} & (I-PC)^{-1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$SM = \left\| \begin{matrix} -(I-PC)^{-1} & (I-PC)^{-1}P \\ -C(I-PC)^{-1} & (I-PC)^{-1} \end{matrix} \right\|_\infty^{-1}$$

Since for good performance and robustness , stability margin must be large number, for this reason “C” must stabilizing controller such that maximize the stability margin. Therefore “C” is derived as following condition that we have:

$$\min_{C} \left\| \begin{matrix} -(I-PC)^{-1} & (I-PC)^{-1}P \\ -C(I-PC)^{-1} & (I-PC)^{-1} \end{matrix} \right\|_\infty \qquad (3)$$

4. *Combination of IMC and FFC for Time Delay System*

Internal Model Controller is an optimum controller which minimizes the effect of disturbance to output signal and considers the uncertainty of control plant and Feed forward controller is an controller which has role of make system’s performance better as much as possible because when time-delay is occurred in system the performance of output signal is poor. Therefore, FFC is suggested to combine with IMC. Also in this research we consider the existence of time-delay elements. Hence most of systems would be an unstable system due to time-delay elements. We suggest the IMC and FFC method to modify the stability of the system by IMC and compensate the performance by FFC. The main reason that we suggest the IMC method is due to considerations of the uncertainty of control plant and time-delay elements. The effects of internal and external disturbance to control plant and output signal are minimized and it can be correspondence to MIMO (Multi Input and Multi Output) and high dimension systems. In order to compensate the stability and performance of system due to time-delay elements, we have designed the hybrid system of IMC and FFC. Figure 3 shows the IMC and FFC system which “du” is internal disturbance to control plant, “dy” is external disturbance to output signal, ”n” is feedback noise, “K1” and “K2” is Internal Model Controller and feed forward controller respectively and "P", "H" and "$\tilde{P}$","$\tilde{H}$","$\tilde{C}$"are actual system, actual time-delay element, model system, approximated and predicted time-delay element and model of loop shape controller, respectively.

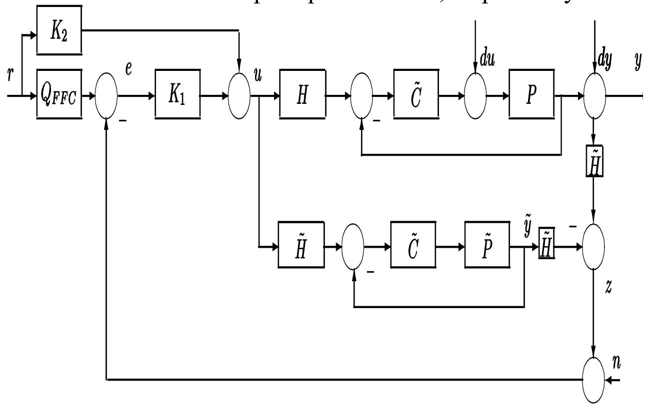

Fig.4: IMC and FFC including a round trip time-delay elements, internal and external disturbance and feedback noise

Through figure 5 the block diagram of IMC and FFC method, we obtained relation between signals and through equations which has shown below, we obtained the complementary sensitivity (4) of figure 4.

$$\begin{cases} y = \Delta P du + \Delta P\tilde{C}Hu \\ \tilde{y} = \tilde{\Delta}\tilde{P}\tilde{H}u \\ z = Hy - \tilde{H}y \\ u = K_2 r + K_1 e \\ e = Q_{FFC}r - (z + n) \end{cases}$$

$$y = \Delta^{-1}P\tilde{C}HM^{-1}\{(K_2 + K_1Q_{FFC})r - n\} + \Delta P\big(I - \tilde{C}HM^{-1}K_1H\Delta P\big)du + \big(I - \Delta P\tilde{C}HM^{-1}KH\big)dy \quad (4)$$

where, $\Delta = \big(I + P\tilde{C}\big)^{-1}, \tilde{\Delta} = \big(I + \tilde{P}\tilde{C}\big)^{-1}, G = H\Delta P\tilde{C}H,\ \tilde{G} = \tilde{H}\tilde{\Delta}\tilde{P}\tilde{C}\tilde{H},\ \Delta G = G - \tilde{G}$ and $M = I + K\Delta G$

Since, in the above equation (4) "dy" multiplied $\big(I - \Delta P\tilde{C}HM^{-1}KH\big)$, IMC method minimizes the effect of disturbance. As a result we consider the minimization of H^2 norm of this coefficient. Equation (5), below, shows how to derive "K_1" which is Internal Model Controller:

$$\min_{K_1}\left\|I - \Delta P\tilde{C}HM^{-1}KH\right\|_2 \quad (5)$$

that is:

$$\frac{\partial}{\partial K}\sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} tr\big[\big(I - \Delta P\tilde{C}HM^{-1}KH\big)^{*}\big(I - \Delta P\tilde{C}HM^{-1}KH\big)\big]d\omega} = 0$$

Theoretically, the optimum Controller "K_1" in equation (6) is calculated as follows:

$$K_1 = \tilde{C}^{-1}\tilde{P}^{-1}(I + \tilde{P}\tilde{C}) \quad (6)$$

where, "K_1" is the inverse system of stabled control plant model system. But note that to realize the controller, if the case of K_1 is an improper system, it is required to add a filter the same as K_1's dimension to make controller proper or strictly proper. The reason of this is for realization of model or controller, system requires to be a proper or strictly proper.

5. *Uncertainty of Control plant and Time Delay Element*

In this case of IMC, we use the model of control plant and the actual control plant which it is an unknown system. So, as we obtained through equation (6) when IMC controller is the inverse system of model, it is optimum case. But, if this model system does not completely match with the actual system, it will be feedback the error between the model system and the actual system. So, the ideal case of this system is when M equal to unit matrix, that is: when $\Delta G = 0$ therefore, M=I.

In this case, the system is called nominal system which is an ideal case. Although in the case of internal model controller except the model system, we have to realize the predicted time-delay element. Therefore, we used the Pade approximation for "$\tilde{L}$" approximated time-delay i.e. time-delay element"$\tilde{H}$" shown as follows:

$$\tilde{H} = Pade(\tilde{L}, n) = e^{-s\tilde{L}} \approx \frac{\sum_{k=0}^{n}(-1)^k c_k \tilde{L}^k s^k}{\sum_{k=0}^{n} c_k \tilde{L}^k s^k} \quad (7)$$

Where, $c_k = \frac{(2n-k)!n!}{2n!k!(n-k)!} \quad (k = 0,1,2,\ldots,n)$

Here we consider the realization of approximated time-delay element as below.

$$\tilde{H} = \left[\begin{array}{c|c} A_{\tilde{H}} & B_{\tilde{H}} \\ \hline C_{\tilde{H}} & D_{\tilde{H}} \end{array}\right] \quad (8)$$

But, remember that the dimension "n" of approximated system transfer matrix must be the same as control plant dimension. The reason of this is that in the multiplication of two system transfer matrix systems, both of them are required to have the same dimension, theoretically. Because of approximated time-delay element is considered as a system transfer matrix.

6. *Simulation and Results*

For evaluating the hybrid system of IMC and FFC method, we have simulated for an unstable control plant with time-delay. Control plant is considered to be 6 dimensional MIMO system which has 2 inputs and outputs.

The process of simulation is to confirm the stability of the closed loop system and compare the performance and stability of IMC and IMC+FFC method with their step response and singular values. Also for IMC+FFC method we have simulated the disturbance response of system and all these processes are simulated for nominal case and non-nominal case.

Simulation conditions

The transfer Matrix of actual system

$$P=\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]=\left[\begin{array}{cccccc|cc}-0.0226 & -36.6170 & -18.8970 & -32.09 & 3.2509 & -0.7626 & 0 & 0\\ 0.0001 & -1.8997 & 0.9831 & -0.0007 & -0.1708 & -0.0050 & 0 & 0\\ 0.0123 & 11.7200 & -2.6316 & 0.0009 & -31.6040 & 22.3960 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -30 & 0 & 30 & 0\\ 0 & 0 & 0 & 0 & 0 & -30 & 0 & 30\\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\end{array}\right]$$

Here we assume that the reference signal has 1 second time-delay to reach the control plant and 1 second to feedback the output signal to reach controller. Therefore, the actual time-delay L=1 and its transfer function is shown as fallows.

$$\mathrm{H}=e^{-sL}I_{2\times 2} \qquad (9)$$

The transfer matrix of model system

$$\tilde{P}=\left[\begin{array}{c|c}\tilde{A} & \tilde{B}\\ \hline \tilde{C} & \tilde{D}\end{array}\right]=\left[\begin{array}{cccccc|cc}-0.0903 & -146.4680 & -75.5880 & -128.3600 & 13.0036 & -3.0503 & 0 & 0\\ 0.0004 & -7.5988 & 3.9325 & -0.0029 & -0.6832 & -0.0199 & 0 & 0\\ 0.0494 & 46.8800 & -10.5264 & 0.0035 & -126.4160 & 89.5840 & 0 & 0\\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -120 & 0 & 15 & 0\\ 0 & 0 & 0 & 0 & 0 & -120 & 0 & 15\\ \hline 0 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 2.5 & 0 & 0 & 0 & 0\end{array}\right]$$

For predicted time-delay element we assume that our estimated time-delay is 0.001 second and to realize the time-delay element in the model system we used 6 dimensional Pade approximation.

$$\tilde{\mathrm{H}}=\mathrm{Pade}(\tilde{L},6)I_{2\times 2} \qquad (10)$$

IMC and FFC proper for nominal case and non-nominal case filters are set up with following conditions:

(1) In non-nominal case IMC and FFC proper filter's parameters are adjustable with uncertainty of time-delay $\Delta L=|L-\tilde{L}|$ and in nominal case with actual time-delay.

(2) Initial values of IMC and FFC proper filters are equal to unit matrix. $Q_{IMC}(0)=I\,,Q_{FFC}(0)=I$

(3) IMC and FFC proper filters required be contain with stable poles.

If system is nominal, in other words $\tilde{P}=P$ and $\tilde{L}=L$

$$Q_{IMC}^{nom}=\begin{bmatrix}\frac{1}{(s^2+3\tilde{L}s+1)^2} & 0\\ 0 & \frac{1}{(s^2+1.5\tilde{L}s+1)^2}\end{bmatrix}=\left[\begin{array}{c|c}A_{Q_{IMC}^{nom}} & B_{Q_{IMC}^{nom}}\\ \hline C_{Q_{IMC}^{nom}} & D_{Q_{IMC}^{nom}}\end{array}\right]$$

$$\overline{Q}^{nom}=\begin{bmatrix}\frac{1}{(s^2+1.7\tilde{L}s+1)^2} & 0\\ 0 & \frac{1}{(s^2+1.5\tilde{L}s+1)^2}\end{bmatrix}=\left[\begin{array}{c|c}A_{\overline{Q}^{nom}} & B_{\overline{Q}^{nom}}\\ \hline C_{\overline{Q}^{nom}} & D_{\overline{Q}^{nom}}\end{array}\right]$$

$$Q_{FFC}^{nom}=(I+Q_{IMC}^{nom})^{-1}\overline{Q}^{nom}=\left[\begin{array}{c|c}A_{Q_{FFC}^{nom}} & B_{Q_{FFC}^{nom}}\\ \hline C_{Q_{FFC}^{nom}} & D_{Q_{FFC}^{nom}}\end{array}\right]$$

Else (non-nominal case)

$$Q_{IMC}^{non}=\frac{\begin{bmatrix}\frac{1}{3.5\Delta Ls+1} & 0\\ 0 & \frac{1}{4\Delta Ls+1}\end{bmatrix}}{(0.01\Delta Ls+1)^3(0.1\Delta Ls+1)}=\left[\begin{array}{c|c}A_{Q_{IMC}^{non}} & B_{Q_{IMC}^{non}}\\ \hline C_{Q_{IMC}^{non}} & D_{Q_{IMC}^{non}}\end{array}\right]$$

$$\overline{Q}^{non}=\frac{\begin{bmatrix}\frac{1}{4\Delta Ls+1} & 0\\ 0 & \frac{1}{3\Delta Ls+1}\end{bmatrix}}{(0.01\Delta Ls+1)^4(0.1\Delta Ls+1)(0.001\Delta Ls+1)}=\left[\begin{array}{c|c}A_{\overline{Q}^{non}} & B_{\overline{Q}^{non}}\\ \hline C_{\overline{Q}^{non}} & D_{\overline{Q}^{non}}\end{array}\right]$$

$$Q_{FFC}^{non}=(I+Q_{IMC}^{non})^{-1}\overline{Q}^{non}=\left[\begin{array}{c|c}A_{Q_{FFC}^{non}} & B_{Q_{FFC}^{non}}\\ \hline C_{Q_{FFC}^{non}} & D_{Q_{FFC}^{non}}\end{array}\right]$$

Here $K_1$and K_2 are IMC and FFC controller respectively which are shown as follows:

$$K_1=\tilde{C}^{-1}\tilde{P}^{-1}(I+\tilde{P}\tilde{C})Q_{IMC} \qquad (11)$$

$$K_2=\tilde{C}^{-1}\tilde{P}^{-1}(I+\tilde{P}\tilde{C})Q_{FFC}=\tilde{C}^{-1}\tilde{P}^{-1}(I+\tilde{P}\tilde{C})(I+Q_{IMC})^{-1}\overline{Q} \qquad (12)$$

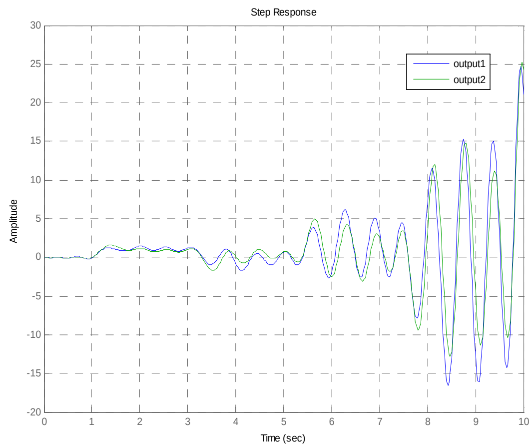

Fig. 5 step response without IMC method

Nominal case

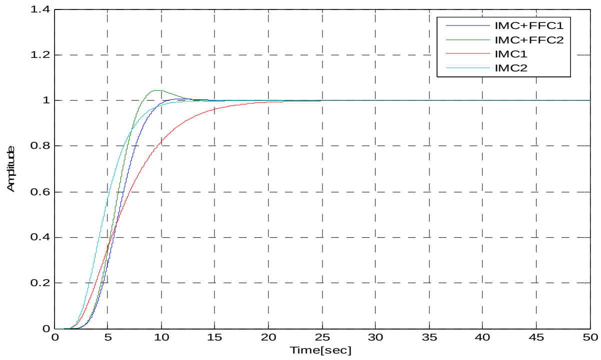

Fig. 6 step response of IMC and IMC+FFC

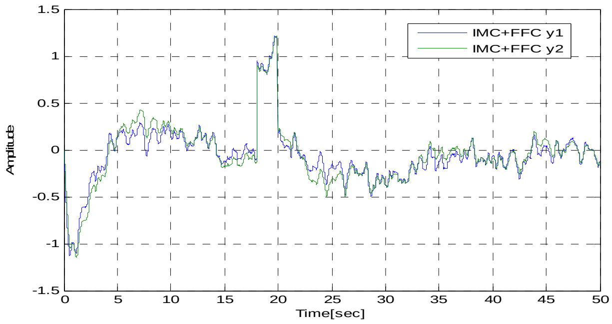

Fig. 7 disturbance response of IMC+FFC

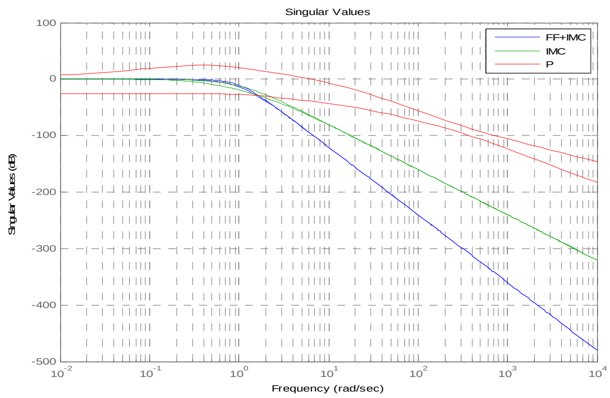

Fig. 8 Singular values of open loop system of plant, IMC and IMC+FFC

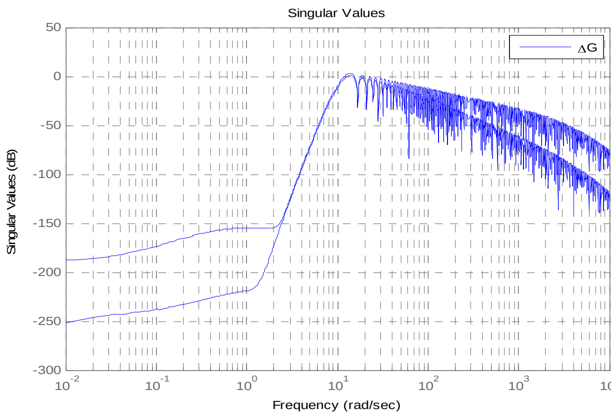

Fig. 9 Singular values of uncertainty system

Non-nominal case

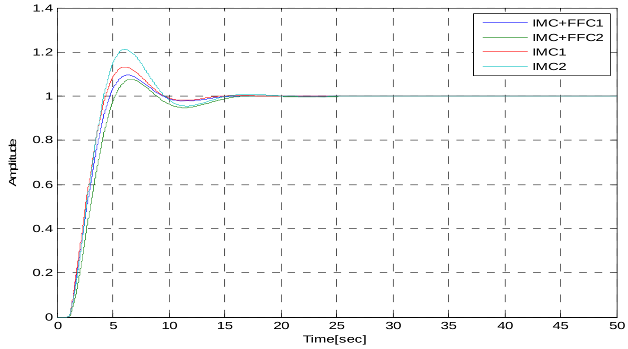

Fig. 10 step response of IMC and IMC+FFC

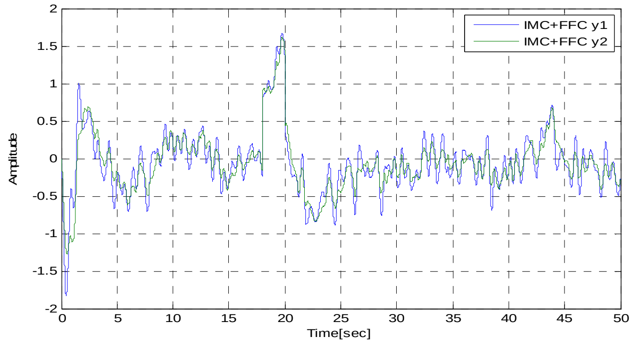

Fig. 11 disturbance response of IMC+FFC

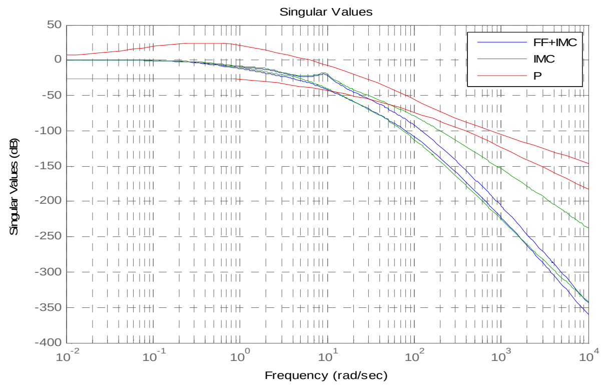

Fig. 12 Singular values of open loop system of plant, IMC and IMC+FFC

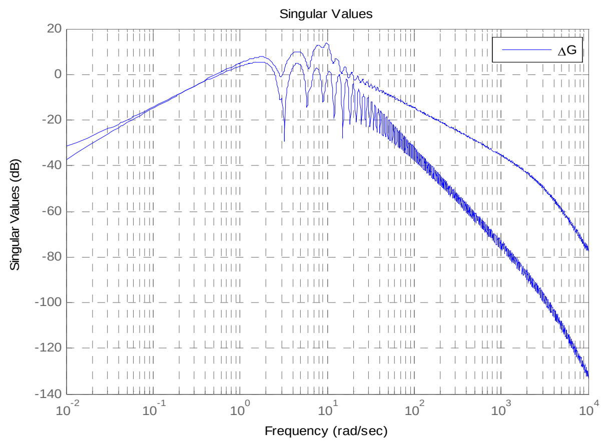

Fig. 13 Singular values of uncertainty system

Results

Through simulation results in Fig. 5 we can see that for time-delay system without IMC method it is unstable. Therefore IMC method is suggested. However in Fig. 6 and Fig. 10 we can confirm the performance of IMC and IMC+FFC for both nominal case and non-nominal case that IMC+FFC has better performance than IMC. For disturbance response of IMC+FFC method it is clear that in non-nominal case the response of system contains with more oscillation than nominal case. This is because of in Fig. 13 the singular values of uncertainty system contains with peek and gain in middle frequencies compare to Fig. 9 the singular values of uncertainty system in nominal case. However the important thing is both in nominal case and non-nominal case in Fig. 8 and Fig. 12 the open loop of controller for IMC and IMC +FFC is shown and it is clear that for both case IMC+FFC method has small gain in high frequencies comparing to IMC. This means that IMC+FFC method is robust to disturbance, noise and uncertainty of control plant and time-delay and it can preserve the stability.

7. Conclusion

In this paper we suggest the IMC+FFC method in order to compensate the performance of the step response and preserve the stability of closed loop system against disturbance and uncertainty. Also in non-nominal case the performance of system is adjustable automatically by uncertainty of time-delay in order to avoid overshoots. However our future works is to design a lower order of controller and preserve the stability against continuously external disturbance.

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